

**Book review**

**Caleb Everett: *Numbers and the Making of Us: Counting and the Course of Human Cultures.*  
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As the existence of this very journal attests, there has been a great change in the range of disciplines that take part in explaining what natural numbers (0, 1, 2, 3, ...) are. For millennia, questions concerning the modalities of existence and knowledge of numbers used to belong almost exclusively to the domain of philosophy. In philosophy, one tradition in particular was dominant for a long time. Starting from at least the Pythagoreans and Plato, it was generally accepted well into modern times that numbers have an objective existence and the way human beings can find out facts about them is by reason, rather than the senses.

In the 19th century, this tradition was challenged by empiricist philosophers like John Stuart Mill (1843), but also by mathematicians who were open to psychological influences, such as Ernst Schröder (1873). However, the momentum that the empiricist and psychologist explanations of numbers could gather was quickly stopped when Gottlob Frege's *Grundlagen der Arithmetik* (1884) gained importance. In that book, Frege made a compelling case that the epistemology of natural numbers should be detached from any psychological considerations and rooted firmly in logical conceptual analysis.

This approach of Frege, further developed by Bertrand Russell (1903), set the tone for much of 20th century discussion on philosophy of mathematics, and that tone was generally resistant to psychological explanations of mathematical objects, including natural numbers. Even empiricist philosophers of mathematics, such as Mill and more recently Philip Kitcher in *The Nature of Mathematical Knowledge* (1983), focused on a *a priori* type of argumentation, engaging almost exclusively in theoretical investigations. While this was the prevalent paradigm in philosophy until the late 20th century, in recent times there has been a visible change in that trend. Currently, increasingly many philosophers are open to including empirical data in their argumentation.

This paradigm change in philosophy is understandable. What used to be the exclusive domain of philosophy now involves many domains of research, including neuroscience, psychology, sociology, anthropology and linguistics. With this new abundance of empirical data, philosophers have also had to reconsider the cognitive foundations of mathematics. This is particularly important because the

data overwhelmingly favor some kind of constructivist position when it comes to natural numbers. Rather than the independently existing abstract objects of Plato - let alone the divinities of Pythagoras - the consensus in different empirical disciplines seems to be that numbers are something human beings have invented, rather than discovered. Consequently, for researchers dealing with empirical studies, there may appear to be limited value in the kind of theoretical *a priori* pursuit that philosophy of mathematics used to be.

Yet, while the multitude of empirical results may show that logical and conceptual analysis of natural numbers is not all there is to the epistemology of natural numbers, philosophical methodology can be useful in interpreting those results, as well as in forming hypotheses and theories. The value of philosophical considerations is already apparent in clarifying terminology, which is necessary for constructing coherent theories. Results concerning infant and animal ability with small numerosities, for example, are too often presented in terms of “numbers”, and the abilities are referred to as “infant arithmetic” or “animal arithmetic”. Presumably, few researchers would be ready to postulate actual arithmetical thinking to infants, yet they see little problem in using arithmetical terminology to describe the infant ability.

In general, it is imperative that the conceptual basis of theories is clarified and there is no conflation concerning the concepts the empirical data concerns. The numerosity concepts infants may have, for example, must be distinguished from the exact notion of natural number. Indeed, at every stage of the development, it must be clear what kind of quantity-concepts are being discussed. Unfortunately this is too often not the case, and it is our contention that several fine works in the field of numerical cognition would have benefitted from a more careful logical and conceptual analysis of the empirical data and the hypotheses presented to explain them. This is also the case with the otherwise highly insightful new book *Numbers and the Making of Us: Counting and the Course of Human Cultures* (2017), by the University of Miami anthropologist Caleb Everett.

The main aim of Everett’s book is to construct a theory about the cultural conditions that have played a key role in the development of numbers. Aside from his own field of anthropology, Everett draws from a wide variety of sources dealing with early human and non-human numerical cognition, and the use of numerals in different cultures. In this way, his work lines up with other accounts aimed at explaining the origins of numbers, counting and arithmetic for the larger public, such as *The Number Sense* by Stanislas Dehaene (2011), *What Counts: How Every Brain is Hardwired for Math* by Brian Butterworth (1999), *Where Mathematics Comes From* by George Lakoff and Rafael Núñez (2000), and *The Universal History of Numbers* by Georges Ifrah (1998).

However, while readers will find large similarities to those books in the way the empirical data are presented, Everett's argumentation differs essentially from all of the above works. Dehaene, for example, focuses on the cognitive systems that form the basis for quantitative information which underlie arithmetical ability. In Butterworth's approach, the main interest also concerns the cognitive tools necessary for mathematics, which he believes humans are inherently equipped with. Everett's main focus, however, is on the cultural importance of *numerals*. As an anthropologist who has written extensively on linguistic issues, this approach makes Everett's book a valuable addition to the literature on numerical cognition.

Throughout the book, Everett's descriptions of anthropological findings are a joy to read. The book is clearly structured and, although anthropology provides the core, it does a wonderful job in presenting data from other disciplines. This makes *Numbers and the Making of Us* a good general introduction to the history of numbers as a human invention, but also to the cognitive basis of numbers. Particularly interesting is Everett's personal angle. As is now commonplace in scientific books intended for the general public, *Numbers and the Making of Us* incorporates parts of travelogue. At times Everett's stories are somewhat detached, but mostly they are highly illuminating. As the son of Daniel Everett, a former missionary who turned into anthropologist when in the Amazon, Caleb Everett has great insight into the numerically limited cultures of Mundurucu and Pirahã. Having spent extensive periods of time in those cultures growing up, Everett has been in a unique position to observe the connections between cognitive, cultural and linguistic differences.

However, what makes Everett's book an important contribution to the current state of knowledge about numerical cognition is that he puts all these anthropological, historical and psychological insights into an original theoretical perspective. The leading idea is that the invention of numbers was the key element that granted many cultures practical advantages that enabled them to establish supremacy over anumeric cultures.

This central claim of Everett is in fact a continuation of two well-established ideas. First, it is often noted that there was an important social change due to trade systems, which could not have happened without symbolic representations of numbers. Second, it is generally accepted that digitalization and automation have had a strong impact on how our contemporary conceptual structures are formed and, in consequence, how the society is structured today. The novel input of Everett is that the social change began at the first introduction of symbolic representations of quantities, perhaps already at the stage of cave art. Trading systems, agriculture, navigation, and later digitalization and automation, were all steps on this general path of how the ability to represent

quantities symbolically made us who we are. In this way, numbers can be compared to other human inventions that have given great advantages in survival and development:

I suggest in this book that a set of conceptual tools called “numbers” - words and other symbols for specific quantities - is a key set of linguistically based innovations that has distinguished our species in ways that have been underappreciated. Numbers are, we shall see, human creations that, like cooking, stone tools, and the wheel, transformed the environment in which we live and evolve. (p. 5)

One of the reasons why the invention of number was so important for the development of the human culture, argues Everett, is that it highly influenced the development of written language. This gives us an important two-way connection between numbers and language. On the one hand, the development of language made symbolic representation of quantities possible. On the other hand, the symbolic representations of quantitative information directly influenced the development of language. Indeed, it is this latter direction that Everett emphasizes:

numbers were quite likely foundational to the advent of writing around the world. It is commonly recognized that the scientific revolution, industrialization, and modern medicine were dependent on specific mathematical practices. Millennia prior to the existence of these practices, though, verbal numerals and inscribed numerals helped enact profound changes in how humans subsisted and how they used symbols to convey ideas [...] In short, spoken numbers and written numerals were pivotal to radical transformations in a variety of cultures millennia ago. In many contemporary endangered cultures similar transformations are at work today. (p. 238)

This way, Everett argues that the importance of inventing symbolic representations of quantities can be traced from the early origins to modern times. Numbers influenced how languages evolved which in turn was crucial in how we developed ideas and tools that gave us the means to survive and thrive in the world, in a way that was not possible for anumeric cultures.

Even though this main thesis is anthropological, to support it Everett brings arguments from various disciplines in social, behavioral, and natural sciences. The arguments are organized in three parts, each of them taking three or four chapters in the book. Part 1 (*Numbers Pervade the Human Experience*) presents the way the invention of numbers has influenced the development of cultures. For this purpose, Everett provides arguments for his view from archaeology and history, but also from the study of numerical cognition and linguistics. Although the importance of numbers for our

culture is well-recognized in the literature, Everett's emphasis goes beyond the usual theories in that in his account, numbers shaped our culture profoundly right from the moment when humans first started using symbolic representations to communicate quantities.

Everett points out how marks and images of hands found in the oldest surviving images, such as the paintings found in the Cosquer and Gargas caves, likely provide the first known symbolic representations of quantitative information. Although we cannot be sure of the intended purpose of the cave paintings, we do know that fingers and hands are generally used in most cultures as first ways to convey quantitative information. In this way, the images of hands may be related to another early form of representing quantities, systems of tallying on stones, bones and pieces of wood. Whereas pictures of hands may have had other purposes, tally marks are difficult to interpret as anything other than representing quantities.

The invention of symbolic representations of quantities was indispensable for representing larger quantities in an exact manner. While we are cognitively equipped to represent non-verbally quantities up to three, or perhaps four, there is no such ability to represent larger quantities exactly. This is well known from research in cognitive sciences, where according to one of the leading paradigms, quantities are implicitly processed in a discrete manner up to four, and then in an approximate manner when the quantities are bigger, at least up to several thousands (Dehaene 2011).

Exact bigger numerals were an important conceptual tool in improving communication for cultures based on agriculture and trade. To store quantitative information, there needed to be a way to write it down. This way, Everett argues, it is likely that numbers were crucial in shaping our languages, written languages in particular. Importantly for Everett, the early cave figures were not only representations of abstract quantities: they were also the first symbolic representations of abstract concepts of any kind. Thus numbers influenced how languages in numerical cultures developed in general, enabling reference to abstract concepts.

In Part 2 (*Worlds without Numbers*), Everett studies how symbolic representations of numbers influenced the way in which human beings structure their representation of the world. He emphasizes the importance of cultures in providing tools to develop optimal environments for surviving. In addition to practical inventions relating to food, safety and comfort of living, Everett points out the importance of conceptual and symbolic tools. The symbolic and conceptual structures provide theoretical frameworks for the development of cultures and have a crucial influence in the way representations of the world develop. In consequence, they play a decisive role in determining the direction in which the cultures further develop also when it comes to practical inventions.

In the case of numbers, the importance of symbolic representations can be seen perhaps most clearly in the still-existing anumeric cultures. The Mundurucu, for example, have only a few number words, and the Pirahã - at least according to Everett's interpretation - do not have any stable words for exact quantities. Whereas cultural isolation has certainly been a factor in them not developing numbers, by now there has been enough contact with numerical cultures for those cultures to adopt numbers. Thus Everett emphasizes the importance of culture-specific conditions. Each culture tends to create and immerse optimal tools for their own circumstances and requirements. In the case of Mundurucu and Pirahã, both hunter-gatherer tribes in the Amazon who do not practice trade extensively, there has been limited use for numerals.

It is easy to accept Everett's view that people growing up in anumeric tribes have different ways of conceptualizing the world than children from numerical societies do. In our culture, children learn to name exact quantities early on, which has an enormous influence on the way they experience the world. While all children have some quantitative abilities at birth, there is a great difference between this natural ability and the culturally developed exact symbolic representations. In addition to studies on infants, Everett also presents results from the research of animal cognition to stress the conceptual difference between treating quantitative information in these different ways.

The idea that language is at the basis of almost all cultural development is explored throughout Everett's book, but it is most extensively developed in Part 3 (*Numbers and the Shaping of Our Lives*). Although Everett emphasizes the way number systems are naturally based on fingers and other body parts, he also notes that using body parts to create number systems has not been a universal development. In this way, he argues that the role of languages, in particular written symbolic representations, has been crucial in developing number systems. Indeed, the characteristics of languages are seen to determine the thinking of its speakers in a fundamental way, both in how they perceive the world and in the inventions they create. Thus Everett is sympathetic toward linguistic relativism, stating that "patterns in language yield patterns in thought" and that "differences between languages can yield differences, often subtle ones, in the cognitive habits of their speakers" (p. 191).

Developing numbers and arithmetic is one such social activity that has been determined by linguistic characteristics. Number words and symbolic numerals, according to Everett, are the kind of conceptual tools that people can learn and communicate easily, and that most people have the motivation to borrow. In the final chapter, Everett formulates his strongest argument for the key role that numbers play in our cultures and cognition:

Only those people who are familiar with number words and counting can exactly differentiate most quantities. The presence of numbers in a language does not just subtly influence how we think about certain quantities, then; it also opens up a door to the world of arithmetic and mathematics. (p. 191)

Presented in this way, there is no mistaking the huge influence Everett gives to the invention of numbers. Our mathematics, indeed our whole way of thinking of the world in terms of quantities, is tied to this invention. It shaped our language, and as such it shaped our cultural progress in a most fundamental way.

As interesting as Everett's analysis of the importance of numbers is, the book is not without its flaws. In the beginning, we claimed that establishing a preconceived philosophical framework would be beneficial for any empirically informed program explaining the acquisition of number concepts. Indeed, this has been done by many of the most prominent researchers in the field of number cognition. Exchanges of ideas between philosophers and cognitive scientists have often proven efficient and fruitful. Worth mentioning in particular is the work of the cognitive scientists Stanislas Dehaene and Elizabeth Brannon, what they call the "Kantian research program". This theoretical program, inspired by a philosophical, neo-Kantian, idea that there exist "a priori intuitions" that precede and structure how humans experience the environment. Dehaene and Brannon write:

Indeed, these concepts are so basic to any understanding of the external world that it is hard to imagine how any animal species could survive without having mechanisms for spatial navigation, temporal orienting (e.g. time-stamped memories) and elementary numerical computations (e.g. choosing the food patch with the largest expected return). (Dehaene & Brannon, 2010, p. 517)

Without explicit reference to Kant, a similar idea is hidden behind the paradigm of core cognition proposed by Elizabeth Spelke (2000) and developed by Susan Carey (2009). It is also highlighted in C. Randy Gallistel's *The Organization of Learning* (1990).

The use of philosophically inspired *object file* (or *mental file*) is another powerful example of confluence of ideas between the two fields. In the late sixties, the concept of mental files was studied in philosophy of mind and language, and since then it has proven to be useful in many philosophical, psychological and linguistic contexts. Francois Recanati (2012), for example, has used David Kaplan's (1989) philosophical theory of indexicals to explain the semantics of integrating information

in terms of mental files. Carey and others have used object files for explaining how the first content from subitizing is represented by a child's cognitive system.

In this way, the conceptual framework of neo-Kantian philosophy and philosophical ideas concerning mental files have influenced the empirical study of numerical cognition. In particular, they have helped the researchers to enhance the clarity of their discourse, as well as enabled them to extend the generality and complexity of their proposals. We feel that Everett's project would also have benefitted from specifying and discussing the philosophical framework to which his work belongs.

As mentioned in the beginning, one important purpose of philosophical methodology is to clarify and systematize the terminology of an area of research. This is particularly pertinent in the subject of numerical cognition and in that respect, Everett is often less than clear. For example, when he talks about recognizing quantities in the subitizing range (one, two, three, four), he refers to this ability as "exact number sense" (p. 105). This may seem like a useful contrast to the standard term "approximate number sense" for the estimation system of larger quantities, but the choice of terminology is highly misleading. There is no consensus among researchers that subitizing is a numerical ability at all, since the representations consists of multiple separate individual object files into which the numerosity is encoded only implicitly (Piazza et al. 2011). But subitizing is certainly not seen as being *exclusively* a numerical ability, which makes the term "exact number sense" for this ability simply confusing.

These types of terminological problems are by no means unique to Everett's work. When it comes to the study of numerical cognition, they are in fact quite common. An example can be found already in the ground-breaking work of Karen Wynn (1992), which Everett explains in length. In Wynn's experiment, infants reacted with surprise to a setting where they saw two dolls put behind a screen, but when the screen was lifted, there was only one doll. Wynn, and Everett following her, argued that the children did the addition  $1 + 1 = 2$  and were surprised when the actual quantity of the dolls did not match the result. Similar results were reported about subtraction. Indeed, the name of Wynn's paper in *Nature* was "Addition and subtraction by human infants".

The experiment has been replicated several times with a lot of variation and since the infants show markers of surprise to a statistically significant degree, there is no reason to doubt that they indeed expected to see two dolls. However, addition and subtraction are arithmetic operations we should associate with exact number concepts. Instead of doing such operations, the surprised reaction may be due to having two separate object files for the two objects. Instead of explaining the infant behaviour by referring to developed arithmetical operations, such as addition and subtraction, we

can explain the same phenomena with much less-developed abilities. This is a crucial distinction when we try to find out how children grasp the addition of natural numbers. If our terminology suggests that we were already doing addition as infants, the explanations will be distorted. Similarly, Everett's term "exact number sense" implies that children can manipulate (small) exact quantities already before they have acquired understanding of numerals or number symbols - which is a bad fit with his general emphasis of numbers as human inventions.

Such confusing use of terminology is a problem in many parts of Everett's book. Already when presenting the fundamental concepts regarding quantities, his treatment would have greatly benefited from conceptual clarity. Everett uses highly unorthodox terminology, using the word *number* for verbal numbers, *numeral* for written numbers, and symbols like 1, 2, 3, 4, ... to refer to "abstract quantities described by numbers" (p. 10). Presumably, numerals describe the same abstract quantities, which makes one think why the distinction between verbal and written numbers is important - especially since both "numbers" and "numerals" are defined by using the term *numbers*.

This might seem like nit-picking, given that it is clear what the difference between verbal and written "numbers" ("numerals" would undoubtedly be a better term for both) is, but in fact there are fundamental problems in Everett's use of these concepts. This can be seen in his analysis of the difference between "quantity" and "number":

It may seem odd to suggest that numbers are a human invention. After all, some might say, regardless of whether humans ever existed, there would still be predictable numbers in nature, be it eight (octopus legs), four (seasons), twenty-nine (days in a lunar cycle), and so on. Strictly speaking, however, these are simply regularly occurring *quantities*. Quantities and correspondences between quantities might be said to exist apart from the human mental experience. (p. 23)

This way, Everett implies that quantities could exist apart from the human mental experience. But what seems like an innocuous conceding of a philosophical possibility turns quickly into something much more problematic. Everett continues on this trail:

Much as color terms create clearer mental boundaries between colors along adjacent portions of the visible light spectrum, numbers create conceptual boundaries between quantities. Those boundaries may reflect a real division between quantities in the physical world, but these divisions are generally inaccessible to the human mind without numbers. (p. 24)

By now there is no doubt that Everett is open to an externalist philosophy when it comes to quantities. We end up with the philosophical position that abstract quantities may exist independently in the world, but human beings need to invent numbers in order to access them. When put in a more standard terminology - that is, natural numbers can have objective existence and we need numerals to refer to them - we see that Everett's theory actually points toward a Platonist philosophy of mathematics.

Platonism, however, is quite an uncomfortable position for Everett's general account. While it is perhaps prudent to leave the externalist position open, together with the naive version of Platonism suggested in the above quotation, it is possible to trivialize his argument as stating that inventing numerals ("numbers" in Everett's terminology) is a crucial step in accessing natural numbers (abstract quantities). But we feel that Everett is arguing for something much more substantial and interesting. That is why presenting the argument in a more carefully constructed conceptual framework would have been highly beneficial. As we see it, Everett is arguing for the position that numerals are a crucial invention in structuring the way we experience the world. Humans have a rough sense of quantity without them, but numerals are necessary for imposing exact, discrete quantities on our experiences. This kind of position may be compatible with Platonism, but as argued in Pantsar (2014), one great strength of the constructivist approach concerning numbers is that there is no need to assume the existence of mind-independent quantities. By suggesting that quantities can reflect "real divisions in the physical world", Everett is needlessly compromising his position.

The matter of conceptual clarity is particularly important because Everett's book has such an ambitious scope. His book is an effort to explain how humans developed from having no numbers to shaping much of their lives around numbers. Such a project is inevitably challenging in terms of conceptual coherence. For example, an important part of the explanation concerns how the number concepts themselves developed. Everett draws from a wide range of sources, but he is not always careful in making the necessary conceptual distinctions that should accompany the data. For example, although his position is that numbers are human inventions, Everett embraces the position that this invention was based on primitive ability with quantities, i.e., subitizing and the approximate number system. This view that the content of number concepts is determined by the primitive abilities is widely shared among cognitive scientists, and also among an increasing number of philosophers. But it has also proven to be a highly challenging problem conceptually. How are the ability to recognize small discrete quantities and the rough estimation ability for larger quantities utilized when creating exact number concepts? Although several promising hypotheses have been presented, the question is still very much open. In Everett's book, however, this subject is not discussed, leaving a

crucial gap in the explanation. We feel that with more careful conceptual analysis this gap would have been visible and the most important turning point for Everett – the invention of symbolic representations for quantities – could have been analyzed more systematically.

The detachment from philosophical literature is particularly troubling in the final chapter of the book (*Transformative tools*), which is thoroughly philosophical in its subject matter. Everett returns to the possibility of Platonism:

While the quantities numbers represent may exist outside our minds, however, the symbolic representations of those quantities are our own innovations, not truly divorceable from our minds. And scientific practices rely on a kind of spiritualization of those anatomically contingent innovations. (p. 235)

Let us compare the treatment of those two sentences. To elaborate on the latter sentence, Everett discusses how particular numbers are often given a higher status than they epistemologically deserve. He points out, for example, how the generally accepted  $p$ -value thresholds of 0.01 and 0.05 (for statistically very significant and significant, respectively) conveniently happen to be multiples of five and ten. In finger-based number systems, Everett argues, there is a natural tendency to set such values as thresholds, even though there is nothing to suggest that, say, 0.02 and 0.06 would not be better choices. This analysis is highly insightful and makes an important point how quantifying something in our culture can often be a clever way to mask arbitrariness.

The first sentence of the above quotation, however, gets the following explanation:

Most people recognize that modern science is based on results that are usually quantified. The scientific method, so tied to math of various sorts, is understandably perceived as the starting point on a path to higher truth. In that sense, atheistic or agnostic proponents of science may also imbue numbers with a kind of spiritual importance, treating them as mind- and body-external realities that guide us toward discoveries of new truths. (p. 252)

Although disguised as a quasi-mystical quip, this quote makes a distinct philosophical claim. It states that the great instrumental value of numbers is responsible for the widely-shared belief that numbers exist independently of us. This would have been a natural point of access to many important questions in philosophy of mathematics, which could have let the reader know just how complex the field of problems involved in such theories is. Instead, we are left with a strong philosophical claim without anything to back it up. One may of course disagree with all the arguments for the mind-

independence of numbers. But surely all the philosophical literature in support of that position cannot be dismissed as spiritually induced delusion in the way Everett suggests.

Ironically, Everett's theory is generally constructed in the kind of systematic way often associated with philosophy. Indeed, this is one of the great strengths of the book. Although it contains an abundance of anecdotes, *Numbers and the Making of Us* rarely succumbs to the kind of impressionistic argumentation that often follows anecdotal evidence. Instead, Everett uses anecdotes to illuminate both the harder empirical data and his own theory of how natural numbers have developed. Most importantly, his theory is successful in pointing out the importance of cultural factors in that development. In recent years, researchers in numerical cognition have emphasized the importance of interdisciplinary collaboration (Alcock et al. 2016). The focus on cultural aspects is a crucial addition to this collaboration, and Everett's work makes an important contribution to the growing field of research on that subject (e.g. Menary 2015, Núñez 2017, Beller et al. *forthcoming*). What we want to suggest is that philosophy should also be included in the interdisciplinary approach. However, as can be seen from Everett's book, there is an important distinction between philosophical topics and philosophical methodology. While *Numbers and the Making of Us* touches upon several philosophical issues, we feel that it does not make sufficient use of philosophical methodology, in particular conceptual analysis.

Nevertheless, it is easy to appreciate the many great strengths of Everett's work. *Numbers and the Making of Us* is meant to work best as an introduction to the topic of how numbers developed and influenced us, and as such it does a commendable job. Our criticism above should at the same time be seen as praise: we believe that the book manages to go beyond being a mere introduction to the empirical data. It makes interesting original arguments concerning the nature of natural numbers based on that data. In doing that, it could have benefitted from a tighter, more systematic conceptual approach. There is much more to the development of numbers than was thought for a long time, and with the current multi-disciplinary work, we are only slowly starting to learn about this development in all its complexity. Everett's book is likely to play an important role in advancing this development.

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