Digital turn and numerical cognition

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Abstract I discuss the influence of effective manipulations of symbols on the conceptual content of the concept of natural number. The process of symbol manipulations is a cultural process and, hence, the conceptual content issued from those manipulations counts as enculturation. Further, I claim that enhancing regularity and automaticity of the process of symbol manipulation by its frequent continual performance, reinforces involvement of the cognitive faculty responsible for regular and computational processes in this concept creation. I consider the contemporary digitalisation of quotidian human life, described by anthropologists and social scientists, to be the reinforcement we are seeking for, the turning point that influenced conceptual shift in the meaning of natural number.

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Introduction

Two analogous problems are discussed in philosophy of computation. The first concerns the ability to overcome skeptical arguments pointing at the impossibility to distinguish between computing and not computing physical devices. The answer to the question why certain objects - such as rocks or plants - are not computing devices, and certain other objects – as computers – are, is far from being straightforward. Piccinini (2017 [16])[1] presents the problem of concrete computations in the following way:

> In our ordinary discourse, we distinguish between physical systems that perform computations, such as computers and calculators, and physical systems that don't, such as rocks. Among computing devices, we distinguish between more and less powerful ones. These distinctions affect our behaviour: if a device is computationally more powerful than another, we pay more money for it. What grounds these distinctions? What is the principled difference, if there is one, between a rock and a calculator, or between a calculator and a computer? Answering these questions is more difficult that it may seem.

The second problem relates vicious circle in the definition of syntactic computability. We assume – which is a common thing to do – that computations relate manipulations of inscriptions. The Church-Turing Thesis states that Turing Machines formally explicate the intuitive concept of computability. The description of Turing Machines requires description of the notation used for the `input` and for the `output`. The notation used by Turing in the original account and also notations used in contemporary handbooks of computability all belong to the most known, common, widespread notations, such as standard Arabic notation for natural numbers, binary encoding of natural

[1] See also, [17].

numbers or stroke notation. The choice is arbitrary and left unjustified. In fact, providing such a justification and providing a general definition of notations, which are acceptable for the process of computations, cause problems. It is so, because the comprehensive definition states that such a notation or encoding has to be computable. Yet, using the concept of computability in a definition of a notation, which will be further used in a definition of the concept of computability yields an obvious vicious circle.

Strangely enough, in the real life we have no problem distinguishing between computing and not computing physical systems. We also have no problem distinguishing a recursive notation for natural numbers from a non-recursive one. However, since both problems are most probably appearances of conceptual fixed points of the conceptual analysis or conceptual engineering [**?**]. Factors that enable us to make these distinctions necessarily come from an external source. In this paper, I explore the possibility of enculturation being such a source.

I start, in **Section 1 (Cognitive enculturation and the number concept)**, by introducing the idea of cognitive enculturation, according to which cultural factors might induce changes in human cognitive structure. Then, I focus on the proposal of Richard Menary (2015 [14]) that number concept acquisition is a paradigmatic example of cognitive enculturation. Menary argues that there are changes at the biological level of brain activity that are activated thanks to symbol manipulation.

After presenting Menary's original account, I suggest, in the **Section 2 (Cognitive basis of the systems of inscriptions)**, two improvements that will enable clarification of where does the intended notation come from, why some notations are recognized as well suited for representing natural numbers, whereas some other are not, *etc.* By "intended notation", I understand a notation that has computational features expected nowadays from natural numbers. However, I also observe, what a notation is expected to model or express, depends on the historical and cultural moment. For instance, contemporary natural numbers in the Western culture, are represented by a recursive sequence of symbols. However, recursivity is a relatively recent feature. Notations based on body parts were certainly not recursive.

I argue that computational features of a contemporary intended notation have a cognitive background. My argument goes as follows. I argue that manipulations of discrete symbols correspond to a specific cognitive faculty, which provides humans with the sensitivity to regularity or the ability to regularly repeat one action (see, Pantsar-Quinon 2015 [15]). Symbols manipulations reinforce cognitive sensitivity to rhythm and provides conceptual content to the concept of computation. Then, in the **Section 3 (Bootstrapping)**, I reformulate Menary's argument in terms of conceptual change. To explain how various conceptual contents, issued from various cognitive backgrounds combine, I use the holistic theory of bootstrapping. In the bootstrapping theory, conceptual contents of various distinct and apparently unrelated concepts get connected. In consequence, conceptual shift at one point, induces conceptual shifts at other points.

The last part of my paper, the **Section 4 (Symbol manipulation and digital turn)**, is devoted to illustrate the increase of the symbol manipulation practice. Not only in purely mathematical contexts, but also in the everyday life, the language issued from overwhelming digitalisation takes over steadily more broader contexts. Sherry Turkle in her books, in particular in "Alone Together: Why We Expect More from Technology and Less from Each Other" (2012 [**?**]), formulates a hypothesis that the language of computing, programming, automation or robotics, evokes real changes in the conceptual structure of the daily language, including ways in which we think of and express our emotions.

1 Cognitive enculturation and the number concept

Cognitive enculturation is one of the positions within the extended cognition paradigm. In this paradigm, one believes that cognitive processes are frequently enhanced by external factors. It opposes the purely scientific way of thinking about cognitive processes based on the assumption that humans are born with all primary cognitive faculties, and that these faculties simply need to mature, or be fine-tuned by learning mechanisms.

The external factors can serve as additional memory resources or supplementary problem solving power. Putnam (1975 [18]) and Burge (1979 [1]) defend a version of externalism where external factors provide support to cognitive faculties, but do not get actively involved in inferential processes. Chalmers and Clark (1998 [4]) speak instead of extended mind, where external factors are involved in reasonings and decision making processes. Even stronger version of externalism is defended by Menary (2015 [14]) for whom external factors can actively participate in changing and developing cognitive structures. In general, the defenders of the theory of cognitive enculturation strongly believe that interactions with the external world influence wiring of our brains.

Menary (2015 [14]) defends a position that he calls "cognitive integration" according to which there exist "multiple cognitive layers", that is "neural, bodily, and environmental processes all conspire to complete cognitive tasks". According to him this kind of frameworks "explains our cognitive capabilities for abstract symbolic thought by giving an evolutionary and developmental case for the plasticity of the brain in redeploying older neural circuits to new, culturally specific functions such as reading, writing, and mathematics" [14, page 2]. The number concept acquisition is the paradigmatic example of how enculturation transforms human cognitive faculties. His argument is formulated within the core cognitive paradigm in numerical cognition (Dehaene 1997/2011 [5], Spelke 2000 [21], Feigenson et al. 2004 [10], Carey 2009 [3], Sarnecka 2015 [20]).

According to the core cognitive paradigm in numerical cognition, humans – and also many non-human animals – are equipped with an innate system or systems that enable them to process quantities without appealing to symbolic representations. There is no full agreement as to which innate factors there are, and which factors amount to the creation of the number concept. Dehaene claims that there exist a cognitive system specialised in processing quantities. The Approximate Number System processes discrete quantitative input and generates continuous representations encoding these quantities in an approximative manner[2].

In the version of the paradigm defended by Carey and Sarnecka the natural number concept is issued in the process of generalisation over first small quantities. The small quantities are apprehended in the process of subitizing, which is a rapid and accurate assessment of a quantity without counting the elements in the collection. Children rely on subitizing to form first conceptual content of the first number words. They first learn to understand that "one" refers to collections with one element, then that "two" refers to collections with two elements, then that "three" means three and "four" means four. At this stage occurs a conceptual leap consisting in grasping a simple version of successor, called "cardinality principle". From this moment on, children are able to associate the number name they know to the given quantity.

When it comes to culturally trigged factors, there exist a common agreement that language grants formation of the concept of extended number. Most famously Hauser, Chomsky and Fitch (2002 [12]) highlight importance of the

[2] There are various versions of how the innate system processing discrete quantities behave, in this paper I will not get involved in this discussion.

language in the process of the number concept acquisition. More specifically, they claim that learning the number concept, along with many other cognitive faculties, depends crucially on the ability to use the language. More specifically, to use the "deep structure" of the language, which is recursivity. Recursivity in their theory is an innate cognitive faculty that structures syntactical layer of the language.

From another perspective, an anthropologist Caleb Everett in his recent book (2017 [9]) claims that symbolic representations of natural numbers, critically changed the direction of cultural development. According to him, the very first tallies – like those in prehistorical caves – opened up the possibility of expressing the abstract concept of natural number, and – most importantly – also abstract concepts, in general. In psychological research – we are going to look at that again later in this paper – Carey (2009 [3]) and Sarnecka (2015 [20]) both claim that learning number names necessarily precedes the process of conceptual content acquisition.

According to Menary (2015 [14]), mathematical cognition underwent – on the phylogenetic level – and undergoes – on the individual ontogenic level – critical changes. The main factor inducing changes at the biological level of brain activity, is symbol manipulation of physical symbolic representations.

Another aspect of the role of mathematical notations is studied in philosophy of mathematical practice in the context of the extended mind thesis, and the use of formalisms and also the use of diagrams.[3] I distinguish the weak and the strong version of the thesis[4]. According to the weak thesis, symbols and diagrams, in general, extend the memory and enable humans to create new concepts and representations. Creation of certain concepts and representations wouldn't have been possible without this external symbolic support, however

[3] About formalisms see works of Catarina Dutilh Novaes *e.g.*, [7], for diagrams of Valeria Giardino, *e.g.*, [11].

[4] Similar idea can be found in Macbeth (2013 [13]).

there is no specific reasons to prefer one notation over another. For instance, daily language provides this extension in a similar way, as does specific and evolved mathematical notation. The latter is simply a handy abbreviation for the specific symbolisms. De Cruz (2008 [6]) highlights the fact that symbol manipulation happens in the public space.

In this paper, I am going to defend the strong version of the thesis, according to which mathematical notation evolves in a specific way and this way depends on is not accidental, but is issued from a specific cognitive constitution of human beings. In the process of mutual reinforcement between features of syntax and cognitive constitution, notation gets increasingly precise not only to express mathematical ideas, but also to fit in the cognitive background. In consequence, this is not any notation that enhances mathematical performance, but a specific set of symbols manipulated in a specific manner over a period of time.

2 Cognitive basis of the systems of inscriptions

In this paper, I claim that Menary's proposal suffers from at least two problems, both – as I will argue – related to the lack of distinction between syntactical and semantical level of the natural number concept. The syntactical level of the natural number concept consists of inscriptions. The semantical level of the natural number concept consists of some sort of abstract objects. In order to explain how the concept of the natural number is acquired (or created), acquisition (or creation) needs to be explained on both layers.

Menary, while discussing core cognitive systems, both naturally and culturally triggered, highlights the fact that:

– First, the ancient system is part of our phylogeny, whereas the discrete system is an acquired set of capacities in ontogeny.

– Second, the ancient system is analogue and approximate, whereas the discrete system is digital and exact.

> The two systems are overlapping but not identical because they have quite different properties. [...Most importantly,] the discrete system operates on symbols that dont map directly onto the ancient system. [14, page 13]

However, he does not explain how this process happens. Menary puts his query outside the standard framework used by cognitive scientists (whos question is how numerical expressions get its meanings where numerical expressions are taken for granted as existing in cultural heritage), but puts all the systems at the same level and in consequence needs to explain where the symbolic representations come from.

The solution that I propose, relies on the idea that there exists a cognitive structure underlying system of symbolic representations. Various versions of this hypothesis can be found in the literature. Hauser, Chomsky and Fitch (2002 [12]) argue that recursivity is an innate cognitive faculty, structuring all linguistic activity. Butterworth (1999 [2]) claims that humans are conditioned to generate mathematics, most interestingly he discusses in this context the appearance of recursive notation for natural numbers.

According to Menary, the human cognitive faculty responsible to process symbolic representations is relatively "new". If there exists a cognitive structure underlying this faculty, then it can be claimed that it is there for one of the two reasons: cultural processes provoked creation of the new faculty or there was conceptual leap with other faculties that has been used for different purposes. In this paper, I oppose this view and I claim that the cognitive faculty responsible to process symbolic representations is an old system, responsible, for example, for sensibility to rhythm that initially might not have amounted to the number concept creation, and that got involved only at some historical or evolutionary point.

My approach enables me to explain why Menarys proposal does not account for historical differences in manipulations of symbolic representations. It is pretty clear that how people manipulate symbols depend on historical moment and the current knowledge of what can be done on the syntactic level. It seems reasonable to suspect that when notation systems for natural numbers were based on body parts, recursivity was not playing any important role. Let me observe that currently operations – that are the most often performed on symbolic representations of natural numbers – are computable functions. This was not the case before the theory of computability got properly formulated in the 1930s. Thanks to relating the concept of natural number with the concept of computation, the concept of natural number received new conceptual content. This conceptual content got particularly intensively spread, because of all sort of computational procedures that ordinary people use in their daily practice.

3 Bootstrapping

The hypothesis formulated in the previous section – that additional cognitive resources started to get involved in the concept creation at some historical point – can be modelled with Carey's theory of bootstrapping. Bootstrapping is a learning theory, according to which the conceptual structure consists of interconnected concepts, forming a conceptual web, mutually influencing each other. Conceptual change, or conceptual enrichment, in one part of the conceptual web, has an impact on concepts in other parts of the web. "Bootstrapping" processes explain how "representational resources that transcend their input can be created" [3, page 305]. In the case of arithmetic, it explains developmental discontinuities observed in the course of the development of the concept of natural number.

Carey's idea is based on the theoretical framework proposed by Quine. Carey translates bootstrapping to more scientific or systematic terms.

To begin with, Carey insists that the process of bootstrapping necessitates explicit symbols.

> The aspect of the bootstrapping metaphor that consists of building a structure while not grounded is applied as the learner initially learning the relations of a system of symbols to one another, directly, rather than by mapping each symbol onto preexisting concepts (Block, 1986). The symbols so represented thus serve as placeholders, at most only partially interpreted with respect to antecedent concepts. [3, page 306]

The placeholder structure provides basis for the conceptual web.

To bootstrap a concept means that this concept gets its meaning thanks to connection between fragmentary, partial or simpler concepts. This aspect of bootstrapping, unfortunately, is not deductive, hence there is no guarantees when it comes to the content of concepts.

> The structures that are tentatively posited either work, in the sense of continuing to capture the observed data that constrain them, or they do not. [3, page 307]

Partial concepts that get involved in bootstrapping a concept can be disclosed only post-factum. There seems to be no rationale favouring one scenario over another. Some just fit together and some do not. What I claim in this paper, is that at least in the case of the concept of natural number the reason which concept will be next influencing its meaning is apt for a full disclosure.

According to Carey number cognition is the prototypical example of bootstrapping.

> The output of the hypothetical bootstrapping mechanism is the numeral list representation of natural number – an ordered list of numerals such that the first one on the list represents 1 and for any word on the list that represents the cardinal value n, the next word on the list represents $n + 1$. The successor function is the heart of numeral list representations of integers. The numeral list representation of number is characterized by Gelman and Gallistels counting principles (the list is stably ordered; individuals in a given count are put in 11 correspondence with number words, and the cardinal value of the set is the ordinal position of the word in the count list).
> The problem of how the child builds an numeral list representation decomposes into the related subproblems of learning the ordered list itself (one, two, three, four, five, six...), learning the meaning of each symbol on the list (e.g., three means three and seven means seven), and learning how the list itself represents number, such that the child can infer the meaning of a newly mastered numeral symbol (e.g., eleven) from its position in the numeral list.

Carey describes two ways in which bootstrapping might lead to the number concept creation. One is based on supervenience over analogue magnitude representations, the second on subitizing of small quantities and then generalisation of the semantical successor function (systematic adding one element to a set). In both bootstrapping scenarios start by assuming that it is possible to learn by heart a list of arbitrary names. She puts numerals, letters from the alphabet and names of days, or months, aside. I claim that numerals differ from other sequences. It is not any arbitrary sequence of names that can be used to represent natural numbers. Funes the Memorious in Borges' novel invents his own system of numerals:

> He told me that in 1886 he had invented an original system of numbering and that in a very few days he had gone beyond the twenty-four-thousand mark. [. . .] In place of seven thousand thirteen, he would say (for example) *Máximo Pérez*, in place of seven thousand fourteen, *The Railroad*; other numbers were *Luis Melián Lafinur*, *Olimar*, *sulphur*, *the reins*, *the whale*, *the gas*, *the caldron*, *Napoleon*, *Augustn de Veida*. In place of five hundred, he would say *nine*. Each word had a particular sign, a kind of mark; the last in the series were very complicated...

The lyrical ego observes:

> I tried to explain to him that this rhapsody of incoherent terms was precisely the opposite of a system of numbers. I told him that saying 365 meant saying three hundreds, six tens, five ones, an analysis which is not found in the "numbers" *The Negro Timoteo* or *meat blanket*. Funes did not understand me or refused to understand me.

As Everett observes in his book, people started manipulating symbolic representations of quantities already in the prehistorical times, when handprints were placed nearby depictions of animals in, let's say the Argentinian Cueva de las Manos. Manipulation of discrete symbolic representations of numbers – and maybe even of any symbols what so ever – conducted humans to observe that certain sequences of symbols have particular properties. For instance, there are sequences that can be generated in an effective manner. The first representations of numbers encoded rather small quantities than recursive principle of successor. Gradually, manipulation of symbols led to contemporary recursive notations that have serviceable computational properties.

Processing regularity is most likely an innate cognitive ability. Since regularity is plainly realised by symbol manipulation and symbols are used to represent discrete quantities, we are dealing with two similar and conceptually overlapping concepts. Connecting them in the bootstrapping process leads to mutual enforcement.

This is where bootstrapping strongly depends on enculturation, and in particular, symbol manipulation.

4 Symbol manipulation and digital turn

Defenders of the weak and strong version of extended cognition usually speak about symbol manipulation in general terms and illustrate their claim with case studies. In this paper, I do not want to formulate any general claims about extended cognition, but instead I want to concentrate on one specific type of manipulation – which gained a lot of importance over the last years – that at some historical point started providing the concept of natural number with an additional conceptual content. As modelled by bootstrapping hypothesis, conceptual contents from different parts of conceptual, and therefore also cognitive, structure, gets correlated and mutually influences. This is, as I claim, what has happened with the concept of natural number and the concept of computation, and it has happened because of a specific type of symbol manipulation, the manipulation of sequences of symbols that got generated in an effective or computable manner.

This type of structured manipulation of symbolic representations happens more and more frequently not only in a mathematical context, but also in the context of other types of reasonings, even in daily contexts. Planning household logistics is guided by structured task assignments, business processes are automated by various methodologies, such as Agile, we write our papers following publication guidelines and our personal work progress ladders. Even my twelve year old son recently declared that he needs to structure his lunch breaks in a more effective manner to be able to spend more valuable time with his various friends (well, that might be a result of dinner conversations with his academic parents).

My claim that the impact of digitalisation increased over last few years is supported – indeed – by anthropological investigations of Sherry Turkle (1984/2004, 2011, 2015). Turkle, a professor at MIT, studies how concepts from computer sciences and robotics get into common language and how they change ordinary peoples approach to inter-personal relations or ethical questions.

According to Turkle the intensity in which digitalisation of the everyday life develops is strongly connected to the fact that computational language was first used to reformulate our perception of our own mind and our consciousness (her earlier work related similar changes that occurred in France in the 1960s and 1970s in consequence of spread of psychoanalytical ideas, see her book "Psychoanalytic Politics: Jacques Lacan and Freud's French Revolution" from 1978 [22]). In "The Second Self : Computers and the Human Spirit" (1984/2005 [23]) Turkle describes these changes that gets into general culture from the digitalisation and robotics in the same way as "psycholanalytic culture" penetrated structures of the general social and political life in France.

> Psychoanalytic language spread into the rhetoric of political parties, into training programs for schoolteachers, into advice-to-the-lovelorn columns. I became fascinated with how people were picking up and trying on this new language for thinking about the self. I had gone to France to study the psychoanalytic community and how it had reinvented Freud for the French taste, but I was there at a time when it was possible to watch a small psychoanalytic community grow into a larger psychoanalytic culture. [23, pages 304–305]

When Turkle speaks about her experience with the digitalised society, she compares the two experiences:

> My experience at MIT impressed me with the fact that something analogous to the development of a psychoanalytic culture was going on in the worlds around computation. At MIT I heard computational metaphors used to think about politics, education, social process, and, most central to the analogy with psychoanalysis, about the self [23, page 305]

She sees in it a first step in the cultural assimilation of a new way of thinking [23, page 305],

> The essential question in such work is how ideas developed in the world of high science are appropriated by the culture at large. In the case of psychoanalysis, how do Freudian ideas move out to touch the lives of people who have never visited a psychoanalyst, people who are not even particularly interested in psychoanalysis as a theory? In the study of the nascent computer culture, the essential question was the same: how were computational ideas moving out into everyday life? [23, page 305]

She searches how "the idea of mind is a program enters into peoples sense of who is the actor when they act" [23, page 305]. A model of mind that is adapted by the society influence how people think about their frustrations and disappointments, their relationships with their families and with their work" [23, page 305]. From the other hand, says Turkle, computer became a new constructed object - "a cultural object that different people and groups of people can apprehend with very different descriptions and invest with very different attributes. Ideas about computers become easily charged with personal and cultural meanings [23, page 308].

In her other books, Turkle studies human attachment to objects. In the volume of essays “Evocative Objects: Things We Think With” she speaks about attachment that people, many of her friends, developed with physical objects. In her book, “Alone Together” Turkle (2011 [25]) extends her observations to different types of automated artificial agents, such as virtual agents mediated by electronic support, or robots. In the series of social experiments, where she asks her subjects to interact with an automated artificial agent, she observes that the stronger attachment develops in the most vulnerable members of our society, such as neglected children with unfulfilled emotional needs, or as old people suffering from the lack of human interactions. Our natural inclination to form emotional attachment with humans, and with objects in the absence of humans, might soon lead to even more human-AI interactions. Those interactions are obviously structured in a very particular, very automated, way, which even more strongly influence digitalisation of the language we use.

5 Conclusions

In this paper, I revisit Menary’s interpretation of the role of enculturation for mathematical cognition, and more particularly, in numerical cognition. According to Menary, the strongest manifestation of enculturation in the case of numerical cognition consists in “the internalisation of the public numeral system [that] allows us to perform the kind of digital mathematical operations that are required for most arithmetic and mathematical operations (Nieder and Dehaene 2009, 197)”. Two systems: the ancient, approximate system and a relatively new and acquired (learned) system for discrete and digital representations and operations overlap.

I suggest a new hypothesis and I claim that the digital turn, that is growing digitalisation of quotidian life, will result (or resulted already) in involving in natural number concept acquisition another cognitive system: the system that makes us sensitive to regularity and which enables us to perform procedures and algorithms. Here I agree with Menary, this system develops and enforces thanks to manipulations of symbols.

In consequence, I propose a slightly different version of enculturation. Instead of searching for new internalised, or external, cognitive capacities to appear, I claim that new cognitive faculties that get involved in the number concept creation relay on of pre-existing cognitive resources that simply provide a conceptual content enrichment.

I conclude that it is plausible that intended notations – notations that we intuitively use to compute and that let us differentiate between computable and non-computable sequence – are the result of the cognitive enculturation process.

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